Approval: 22nd Senate Meeting

Course Name	: Functional Analysis
Course Number	: MA-521
Credit	: 3-1-0-4
Prerequisites	: MA-511 (Real Analysis)
Students intended for	: M.Sc./B. Tech./M.S./Ph.D.
Elective or core	: Core for M.Sc. in applied Mathematics and Elective for other discipline.
Semester	: Odd/Even

Preamble: The objective of this course is to introduce fundamental topics in Functional Analysis. The contents are designed in such a way that it will give foundation of Functional Analysis at a level and depth appropriate for someone aspiring to study higher level mathematics. Many of the concepts and results like Hahn Banach, open mapping, closed graph theorems etc are very useful in proving many results in fields like Differential Equations, Numerical Analysis etc.

Module 1: Normed spaces, Examples of Normed Spaces, Subspaces of Normed Spaces, Quotient Normed Spaces, Riesz Lemma, Finite-Dimensional Normed Spaces, Convex Subsets of Normed Spaces, Stronger and Equivalent Norms, Strictly Convex Normed Spaces. [5 hours]

Module 2: Linear Maps Between Normed Spaces, Continuity of linear maps, Examples of Discontinuous Linear Maps on Infinite Dimensional Normed Spaces, Various Criterion for Continuity of Linear Maps, Linear Functionals, Examples of Continuous Linear Maps, Necessary Conditions for the Continuity of Transformations defined by Infinite Matrices, Operator Norm of Bounded Linear Maps, Operator Norm of Transformations defined by Finite Matrices. [5 hours]

Module 3: Hahn-Banach Separation Theorem, Hahn-Banach Extension Theorem, Consequences of Hahn-Banach Extension Theorem, Uniqueness of the Hahn-Banach Extension, Banach Limits.[5 hrs]

Module 4: Banach Spaces, Subspaces of Banach Spaces, Quotient Banach Spaces, Product of Banach Spaces, Canonical Embedding of Normed Spaces, Schauder Basis, Uniform Bounded Principle and its Applications, Banach-Steinhaus Theorem. [8hours]

Module 5: Closed Maps, Closed graph theorem, Linear Projections, Open Maps, Quotient Maps, Open Mapping Theorem and its Applications, Bounded Inverse Theorem. [6hours]

Module 6: Spectrum of Bounded Operators, Resolvent Set, Eigen-spectrum, Approximate Eigen-spectrum, Spectrum of the Right Shift Operator, Compact Operators on Normed Spaces, Spectrum of Compact Operators. [5 hours]

Module 7: Inner Product Spaces, Orthonormal Sets, Bessel's Inequality, Riesz-Fischer Theorem, Fourier Expansion, Parseval Formula, Projection and Riesz Representation Theorems, Bounded Operators and Adjoints, Normal, Unitary and Self-Adjoint Operators. [8 hours]

Text Books:

1. **B.V. Limaye, Functional Analysis, New Age International Private Limited, Revised Third Edition, New Delhi, x + 612 pp., 2017.**
2. **B.V. Limaye, Linear Functional Analysis for Scientists and Engineers, Springer, Singapore, xiv + 254 pp., 2016.**
3. **J.B. Conway, A Course in Functional Analysis, Springer, 2nd Edition, 1990.**

Reference Books:

1. **E. Kreyzig, Introductory Functional Analysis with Applications, John Wiley & Sons, New York, 1989.**
2. **K. Yoshida, Functional Analysis, Springer, 6th Edition, 1995.**
3. **C. Goffman and G. Pedrick, A First Course in Functional Analysis, Prentice-Hall, 1974.**
4. **A. Taylor and D. Lay, Introduction to Functional Analysis, Wiley, New York, 1980.**